5.5 – Building Inverses of Functions

Daily Objectives:

1. Define the inverse relation of a function.
2. Given several points in a function, find inverse points.
3. See relation-inverse symmetry across the line $y = x$
4. Find the composition of functions with their inverses
5. Apply inverses in real-world situations

> **Inverse of a Relation**
>
> You get the **inverse of a relation** by exchanging the x- and y-coordinates of all points or exchanging the x- and y-variables in an equation.

Graph the equation $f(x) = 6 + 3x$ on your calculator. Complete the table for this function:

X	-2	-1	0	1	2	3
Y	0	3	6	9	12	15

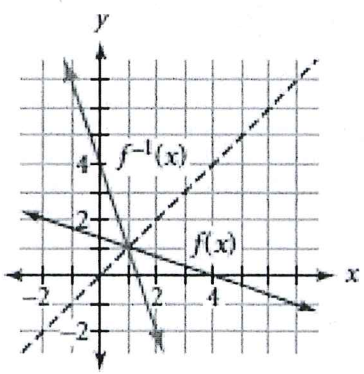
Because the inverse is obtained by switching the independent and dependent variables, you can find five points on the inverse of function *f* by swapping the x- and y-coordinates in the table. Complete the table for the inverse:

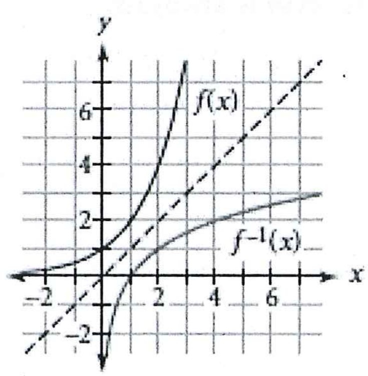
X	0	3	6	9	12	15
Y	-2	-1	0	1	2	3

What is the equation for the points in the table above?

$y = \frac{1}{3}x - 2$

Graph each equation on your calculator. What do you notice about the graphs?





A function and its inverse are reflections across the line $y = x$.

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Example 1: Find the inverse equation:

a. $f(x) = 6 + 3x$

$x = 6 + 3y$

$-6 \quad -6$

$\frac{x-6}{3} = \frac{3y}{3}$

$y = \frac{1}{3}x - 2$

$\boxed{f^{-1}(x) = \frac{1}{3}x - 2}$

b. $g(x) = \sqrt{x+1} - 3$

$y = \sqrt{x+1} - 3$

$x = \sqrt{y+1} - 3$

$x + 3 = \sqrt{y+1}$

$(x+3)^2 = y + 1$

$(x+3)^2 - 1 = y$

$\boxed{f^{-1}(x) = (x+3)^2 - 1}$

c. $h(x) = (x-2)^2 - 5$

$y = (x-2)^2 - 5$

$x = (y-2)^2 - 5$

$x + 5 = (y-2)^2$

$\pm\sqrt{x+5} = y - 2$

$2 \pm \sqrt{x+5} = y$

$\boxed{f^{-1}(x) = 2 \pm \sqrt{x+5}}$

One-to-One: When an equation and its inverse are both functions.

Are any of the equations from example 1 one-to-one?

a & b

Inverse of c Fails Vertical Line Test

Original fails horizontal line test

Example B: Use the equation below to find the following:

a. Find $f^{-1}(x)$: $f(x) = 4 - 3x$

$y = 4 - 3x$

$x = 4 - 3y$

$\frac{x - 4}{-3} = \frac{-3y}{-3}$

$-\frac{1}{3}x + \frac{4}{3} = y$

$\boxed{f^{-1}(x) = -\frac{1}{3}x + 1\frac{1}{3}}$

b. $f^{-1}(f(x))$

$= -\frac{1}{3}(4 - 3x) + 1\frac{1}{3}$

$= -\frac{4}{3} + x + 1\frac{1}{3}$

$f^{-1}(f(x)) = x$

c. $f(f^{-1}(x))$

$= 4 - 3\left(-\frac{1}{3}x + \frac{4}{3}\right)$

$= 4 + x - 4$

$f(f^{-1}(x)) = x$

The composition of a function and its inverse is always *x*.